KC-6165

B. E. - II (Sem. III) (Comp. / I.T.) Examination

November / December – 2012

Discrete Mathematics

Time : 3 Hours] [Total Marks : 100

Instructions :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fillup strictly the details of ☛ signs on your answer book.

Name of the Examination :

☛ **B. E. - 2 (SEM. 3) (COMP. / I.T.)**

Name of the Subject :

☛ **DISCRETE MATHEMATICS**

☛ Subject Code No. : **6 1 6 5** ☛ Section No. (1, 2,.....) : **NIL**

Seat No. :

Student's Signature

(2) Attempt all questions.

(3) Figures on right indicate full marks.

1 (a) Do as directed **10**

(1) Define Euler graph.

(2) Define complete graph.

(3) Define unicursal line and unicursal graph.

(4) Define a binary tree.

(5) Define a cut set.

(b) Attempt any two **10**

(1) Define degree of a vertex. prove that the number of odd vertices in a graph is always even.

(2) If a graph has exactly two vertices of odd degree, then prove that there must be a path joining these two vertices.

(3) Prove that a given connected graph G is an Euler graph, if and only if all vertices of G are of even degree.

2 (a) Describe Konigsberg bridge problem . **3**

(b) Attempt any **three** : **12**

(1) Define a spanning tree. prove that a pendent edges in a connected graph G is contained in every spanning tree of G.

(2) Prove that in a complete graph with n vertices, there are $\left(\frac{n-1}{2}\right)$ edge -disjoint Hamiltonian circuits, if n is an odd number ≥ 3.

(3) Define a path. If the intersection of two paths is a disconnected graph, prove that the union of the two paths has atleast one circuit.

(4) Prove that a graph is a tree if and only if it is minimally connected.

3 (a) Describe three utilities problems. **3**

(b) Attempt any **three** : **12**

(1) Define a planner graph. Prove that a connected planner graph with n vertices and e edges has $e-n+2$ regions.

(2) Define distance between two spanning trees of a connected graph G. In usual notations, show that $\max d\left(T_i, T_j\right) \leq \min(\mu, r)$, where μ and r are nullity and rank of a graph G respectively.

(3) Describe Kuratowski's two graphs.

(4) Prove that a vertex V in a connected graph G is a cut-vertex if and only if there exist two vertices x and y in G, such that every path between x and y passes through V.

4 (a) Do as directed **10**

(1) Define transitivity of a binary relation. Show whether relation R defined as

$R = \{(1,2),(2,3),(1,3),(2,1),(1,1)\}$

is transitive or not.

(2) Draw Hasse diagram of (S_{30}, D), where S_{30} is the set of divisors of 30 and D stands for divisor.

(3) Define an equivalence relation.

(4) Define connectives NAND and NOR.

(5) What can you say about sets A and B if given that

(i) $A \cup B = \varphi$

(ii) $A - B = A$

(b) Attempt any two : **8**

(1) Obtain $P \Leftrightarrow Q$ in disfunctive normal form.

(2) Let $S = \{a, b, c\}$ and $A = \rho(s)$, power set of A. Draw the Hasse diagram of the poset A with the partial order $\subseteq$ (set inclusion).

(3) Let f be a function from $N \times N$ to N defined as

$f((x, y)) = x + y, x, y \in N$

What can you say about one to one ness and onto ness of f ?

5 (a) Attempt any **two** : **8**

(1) Show that a lattice with three or fewer elements is a chain.

(2) Show that in a lattice if a $a \le b \le c$ then $a \oplus b = b * c$ and $(a * b) \oplus (b * c) = b = (a \oplus b) * (a \oplus c)$.

(3) Show that $\langle S_{30}, D \rangle$ is a complemented lattice.

(b) Attempt any **two** : 8

(1) Define an abelian group. Show that if every element in a group is its inverse, then the group must be abelian.

(2) Prove that the kernel of a homomorphism g from a group $(G,*)$ to (H,Δ) is a subgroup of $(G,*)$.

(3) Prove that the order of a subgroup of a finite group divides the order of the group.

6 (a) Attempt any **two** : 8

(1) Show that in a complemented, distributive lattice $a \le b \Leftrightarrow a*b' = 0$.

(2) Prove that every chain is a distributive lattice.

(3) In any boolean algebra, show that

(i) $a+(a'b)=a+b$

(ii) $a(a'+b)=ab$

(b) Attempt any **two** : 8

(1) Define primitive recursive function. Let $\left[\sqrt{x}\right]$ be the greatest integer $\le \sqrt{x}$. Show that $\left[\sqrt{x}\right]$ is primitive recursive.

(2) Define recursive function show that the set of divisors of a positive integer n is recursive.

(3) Use the karnaugh map representation to find a minimal sum of products expression of the function $f(a,b,c,d)=\sum(0,1,2,3,13,15)$.